Analytical design and analysis of mismatched Smith predictor

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Abstract

In this paper, an analytical design method is developed for the mismatched Smith predictor on the basis of the internal model control (IMC) method. Design formulas are given and design procedure is significantly simplified. One important merit of the proposed method is that the response of the closed loop system can be easily adjusted. The relation between the controller parameter and the system response is monotonous. In addition, necessary and sufficient condition for the robust stability of mismatched Smith predictor is also given. It is shown that the proposed method can provide good performance for perfectly matched system and a better response for mismatched system. Several numerical examples are given to illustrate the proposed method. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Synthesis and tuning of control structures for single input–single output (SISO) systems comprises the bulk of process control problems. In the past, hardware considerations dictated the use of PID controllers [1], but through the use of computers, controllers have now advanced to the stage where virtually any conceivable control policy can be implemented. Due to the progress, the design of Smith predictor [2] is widely studied in the past decades. Though the Smith predictor offers potential improvement in the closed loop performance over conventional controllers, it suffers from a sensitivity problem. In the face of inevitable mismatches between the model and actual process, the closed loop performance can be very poor [3]. A lot of work has been done in relation to the robustness issues of the Smith predictor system. For example, [4] systematically studied the robust performance of the Smith predictor within the IMC structure using several design methods; [5] presented a simple criterion for the tuning of Smith predictor when the plant time delay is not precisely known; [6] discussed robust PID tuning problem for the Smith predictor in the presence of model uncertainty; and [7] developed a two degree-of-freedom robust Smith predictor. Recently, Wang et al. [8] proposed a new scheme. It employs a deliberately mismatched model to enhance performance over a perfectly matched system while using a simple primary controller.

Based on the IMC theory, this paper will develop an analytical design procedure for the mismatched Smith predictor proposed by [8]. For controller design, simplicity, as well as optimality, is important. Compared with the empirical method of [8] the analytical method can give design formulas and thus simplified the design procedure. On the other hand, the result of IMC is suboptimal. This provides possibility of improving the closed loop performance.

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The paper is organized as follows. In the following section the assessment of the achievable performance of classical Smith predictor is formulated. In the third section an equivalent representation of the mismatched Smith predictor is given and the design formulas are derived analytically. The robustness is also discussed. Several examples are provided in the fourth section, in which the new controller is compared with that of [8]. Finally, conclusions are given in Section 5.

2. Assessment of achievable performance

The structure of classical Smith predictor is shown in Fig. 1, where \( R(s) \) is the controller, \( G(s) \) is the actual plant, \( G_m(s) \) is the model, and \( G_{m_0}(s) \) is the delay free part of \( G_m(s) \). The closed loop transfer function between the setpoint \( r(s) \) and the output \( y(s) \) is

\[
T(s) = \frac{R(s)G_m(s)}{1 + R(s)(G_{m_0}(s) - G_m(s) + G(s))} \quad (1)
\]

In the case of perfect modeling, i.e. \( G_m(s) = G(s) \), the closed loop transfer function is given by

\[
T(s) = \frac{R(s)G_m(s)}{1 + R(s)G_{m_0}(s)} \quad (2)
\]

This implies that the characteristic equation is free of the time delay so that the primary controller \( R(s) \) can be designed with respect to \( G_m(s) \). The achievable performance can thus be greatly improved over a conventional system without delay compensation.

The Smith predictor can be related to IMC structure, which has been discussed by [9] and a more strict proof was given by [10]. Define the IMC controller

\[
Q(s) = \frac{R(s)}{1 + G_{m_0}(s)R(s)} \quad (3)
\]

Then the two schemes are equivalent to each other. The closed loop transfer function can be written as

\[
T(s) = G_m(s)Q(s) \quad (4)
\]

The primary objective of any feedback control scheme is to make the difference between the controlled outputs \( y(s) \) and desired setpoint \( r(s) \) as small as possible. What is meant by “small” can be defined in terms of various performance specifications for the closed loop system. In IMC method, the performance objective is defined as \( H_2 \) optimal, i.e. min \( \int (r(t) - y(t))^2 dt \). Assume that the plant is stable. Factor the model

\[
G_{m_0}(s) = G_{m_0}^+(s)G_{m_0}^-(s) \quad (5)
\]

such that \( G_{m_0}^+(s) \) contains all right half plane zeros and is all pass. The suboptimal IMC controller can be expressed as

\[
Q(s) = J(s)/G_{m_0}^-(s) \quad (6)
\]

where \( J(s) \) is a user-specified filter. Let \( m \) be the relative degree of \( G_{m_0}^+(s) \). The filter with asymptotic tracking ability is usually selected as

\[
J(s) = 1/(\lambda s + 1)^m \quad (7)
\]

Here \( \lambda \) is a positive real constant. The controller of Smith predictor can then be obtained through

\[
R(s) = \frac{Q(s)}{1 - G_{m_0}Q(s)} \quad (8)
\]

By tuning \( \lambda \), one can adjust the nominal performance and robust performance monotonously. In the case of perfect match, the nominal performance can arbitrarily approach optimality by decreasing \( \lambda \).
3. Analytical design of modified Smith predictor

Let the delay free part of the model be
\[ G_m(s) = \frac{K_0}{(\tau_0s + 1)^n}, \quad n = 1, 2, 3 \] (9)

where \( n \) is specified by the user. The default value of \( n \) is suggested to be 2 by [8]. Factor it into
\[ G_{m0}(s) = G_{m01}(s)G_{m02}(s), \quad \text{where} \]
\[ G_{m01}(s) = \frac{K_0}{\tau_0s + 1} \] (10)
\[ G_{m02}(s) = \frac{1}{(\tau_0s + 1)^{n-1}} \] (11)

An equivalent of the mismatched Smith predictor proposed by [8] is shown in Fig. 2.

Though a certain uncertainty is introduced by the simplified model \( G_m(s) \) in the scheme, we can also design it by IMC method. Regard \( G_m(s) \) as the nominal plant. The IMC controller is
\[ Q(s) = \frac{(\tau_0s + 1)^n}{(\lambda s + 1)^n} \] (12)

On the other hand,
\[ Q(s) = \frac{R(s)}{1 + G_{m01}(s)R(s)} \] (13)

Therefore, the controller is derived analytically as follows
\[ R(s) = \frac{(\tau_0s + 1)^n}{(\lambda s + 1)^n - (\tau_0s + 1)^{n-1}} \] (14)

which is a PID controller for the default value of \( n \)
\[ R(s) = \frac{(\tau_0s + 1)^2}{\lambda^2s^2 + (2\lambda - \tau_0)s} \] (15)

Furthermore, apart from the introduced certain uncertainty there always exists other uncertainties in practice. Assume that
\[ \left| \frac{G(s) - G_m(s)}{G_m(s)} \right| = \left| \frac{\epsilon(s)}{G_m(s)} \right| \leq \tilde{\epsilon}(\omega) \]

where \( \tilde{\epsilon}(\omega) \) is the bound on the multiplicative uncertainty \( \epsilon(\omega) \) (it consists of certain uncertainty and other uncertainties). The nominal transfer function of the closed loop system is
\[ T(s) = \frac{G_m(s)R(s)}{1 + G_{m01}(s)R(s)} \] (16)

Then the closed loop system is robustly stable if and only if [9]
\[ \left| \frac{G_m(\omega)R(\omega)}{1 + G_{m01}(\omega)R(\omega)} \right| \leq \frac{1}{\tilde{\epsilon}(\omega)}, \quad \forall \omega \] (17)

This implies that there is a low bound larger than zero for \( \lambda \) in a uncertainty system.

4. Examples

Three typical plants from [8] will be used in this section to illustrate the proposed method. The perfectly matched system is studied in example 1. For comparison, the mismatched cases are studied in example 2 and 3 for both the proposed method and Wang’s method. Since \( \lambda \) has a monotonous relation with the response of the closed loop system, one can easily get the required response.

- Example 1 — Consider the following process [8]
\[ G(s) = \frac{e^{-4s}}{(s + 1)^3} \]

Take \( \lambda = 0.1, \lambda = 0.5 \) and \( \lambda = 1 \), respectively, for the proposed Smith predictor. When the system is
perfectly matched, we have

\[ Q(s) = \frac{(s + 1)^5}{(\lambda s + 1)^2} \]

\[ R(s) = \frac{(s + 1)^5}{(\lambda s + 1)^2 - (s + 1)^4} \]

A unit step setpoint change is introduced at \( t = 0 \), and a 10% step disturbance is introduced at the plant input at \( t = 80 \). It is seen that the proposed method can provide arbitrarily good performance by decreasing \( \lambda \) (Fig. 3). As a matter of fact, for perfectly matched case, if \( \lambda \) tends to be zero, the closed loop transfer function has an infinity bandwidth and thus the performance tend to be optimal [9]. When there exists uncertainty, the sufficient and necessary condition (17) shows that the smaller \( \lambda \) is the worse the robustness is. In this case the designer should increase \( \lambda \) from a very small positive real monotonously until a satisfied trade-off between performance and robustness is obtained.

- Example 2 — Suppose that the process is given by

\[ G(s) = \frac{1}{(s + 1)^{10}} \]

The simplified process model is computed as [8]

\[ G_m(s) = \frac{1}{(2.43s + 0.995)^2} e^{-5.39s} \]

and Wang’s controller [8] is

\[ R(s) = \frac{6.25s + 2.56}{s(s + 2.29)} \]

Take \( \lambda = 0.6667 \) and \( \lambda = 1 \) for the proposed Smith predictor. We have

\[ Q(s) = \frac{(2.43s + 0.995)^2}{(\lambda s + 1)^2} \]

\[ R(s) = \frac{(2.43s + 0.995)^2}{\lambda^2 s^2 + (2\lambda - 2.43)s} \]

A unit step setpoint change is introduced at \( t = 0 \), and a 10% step disturbance is introduced at the plant input at \( t = 60 \). The closed loop responses of the two systems are shown in Fig. 4. It is seen that for \( \lambda = 0.6667 \) the proposed method provides a better response than that of [8] and for \( \lambda = 1 \) a little better response is obtained.

- Example 3 — Consider a non-minimum phase process with time delay [8]

\[ G(s) = \frac{-s + 1}{(s + 1)^2} e^{-2s} \]

![Fig. 3. Performance of perfectly matched system (Solid line: \( \lambda = 0.1 \), dotted line: \( \lambda = 0.5 \), dashed line \( \lambda = 1 \)).](image-url)

![Fig. 4. Multiple lag process (Solid line: \( \lambda = 1 \), dotted line: \( \lambda = 0.6667 \), dashed line Wang’s method)).](image-url)
The mismatched model is given by [8]
\[ G_m(s) = \frac{1}{(1.46s + 0.999)^2} e^{-5.07s} \]
and Wang’s controller [8] is
\[ R(s) = \frac{5.85s + 4}{s(s + 2.83)} \]
Take \( \hat{\lambda} = 0.6667 \) and \( \hat{\lambda} = 1 \) for the proposed Smith predictor. Therefore,
\[ Q(s) = \frac{(1.46s + 0.999)^2}{(\hat{\lambda}s + 1)^2} \]
\[ R(s) = \frac{(1.46s + 0.999)^2}{\hat{\lambda}^2s^2 + (2\hat{\lambda} - 1.46)s} \]
A unit step setpoint change is introduced at \( t = 0 \), and a 10% step disturbance is introduced at the plant input at \( t = 40 \). The closed loop responses for both cases are presented in Fig. 5. Again a performance improvement is achieved for \( \hat{\lambda} = 0.6667 \) with the proposed method and a similar response is obtained for \( \hat{\lambda} = 1 \).
The usefulness of the method is not only in performance enhancement, but also lie in that the designer can easily adjust the nominal performance and robust performance by \( \hat{\lambda} \) and thus make a tradeoff between the two conflicting objectives.

5. Conclusions

Several problems are discussed in this paper:
1. It has been shown that for perfectly matched model, the performance of the closed loop system can arbitrarily approach the optimal. Thus, it is not necessary to utilize the mismatch for improving the system performance.
2. Practical plants are of high order and models are usually low order. To improve the system performance, [8] presented a mismatched Smith predictor structure. It is found that the structure can be related to the IMC. A new design procedure is then developed in the framework of IMC theory and analytical results are provided. Simulations show that improved response is obtained.
3. Necessary and sufficient condition for robust stability is provided for the mismatched Smith predictor system. If the exact uncertainty profile is known, the exact controller parameter can be calculated. In the context of process control, one can adjust the performance and robustness of the closed loop system in such a way: increase the controller parameter from a small positive real monotonously until a satisfactory response is obtained.

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References

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