Robust digital controller design for processes with dead times: New results

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Abstract: The Dahlin controller is studied in the complex-frequency domain in terms of performance and robust stability. Several well known digital control algorithms are compared with the Dahlin controller and found to be equivalent to it. The possibility of extending the Dahlin controller to the control of plants with zeros outside the unit circle and unstable plants is discussed. The essential cause of ringing is investigated and some ambiguous formulation is clarified. A new procedure is developed for digital controller design. Compared with conventional methods, it provides a more effective method of eliminating ringing. Finally, numerical examples are given to illustrate the new approach.

1 Introduction

The Dahlin controller is a distinctive algorithm for the control of single input/single output (SISO) plants with dead time. It was proposed by Dahlin [1] and Higham [2], independently. The attractiveness of this design technique comes from the fact that it is simple to apply and can give an acceptable performance. It has been studied by various researchers and included in many current textbooks [3–8]. Astrom and Wittenmark [9] point out that the Dahlin controller is a controller of the Smith predictor type [10], while Zhang and Sun [11, 12] find that if the optimal/suboptimal Smith predictor for a first-order plant with dead time is sampled, it is identical to the Dahlin controller. In this paper, the Dahlin controller is explained in the complex-frequency domain by comparing it with some seemingly different design techniques. An analysis of the performance and robust stability is also provided. This initial work gives an insight into the merits of the Dahlin controller.

Usually there are two problems associated with the Dahlin controller [13]: Firstly it cannot be used for the control of plants with zeros outside the unit circle and unstable plants, and secondly ringing exists in the controller output, which may cause unnecessary equipment wear.

The first problem is still unresolved. For the second problem, Dahlin [1] has proposed a modification, but it seems that the modified method does not eliminate ringing completely. By modifying the closed-loop transfer function, Zhang [14] developed a new design technique to eliminate ringing, but it leads to the sacrifice of the system’s performance. For both methods, ‘An additional disadvantage is that we do not know beforehand for which systems problems will appear’ [13].

In this paper, several well known digital control algorithms are compared with the Dahlin controller and found to be equivalent to it. Based on the discussion, the Dahlin controller is extended to the control of plants with zeros outside the unit circle and unstable plants. The real cause of ringing in sampling systems is analysed, and a new procedure is developed for digital controller design. It is shown that the new method is more effective than conventional ones.

![Figure 1: Closed-loop system with unity feedback](image)

2 Performance and robust stability

Consider the unity feedback control loop shown in Fig. 1, where \( C(s) \) is the controller and \( G(s) \) is the plant. As the Dahlin controller is designed for first-order plants with dead times, the discussion of the paper is also limited to this kind of plant. Assume that

\[
G(s) = \frac{Ke^{-\theta s}}{Ts+1}
\]

and the system input is a unit step. The Dahlin controller is designed by specifying the closed loop transfer function \( T(s) \) to be a first-order process with dead time equal to that of the plant \( G(s) \), i.e.

\[
T(s) = \frac{e^{-\theta s}}{\lambda s + 1}
\]

where \( \lambda \) is the time constant of the closed-loop response. On the other hand, the closed-loop transfer function can be described as

\[
T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}
\]
This relationship can be rearranged to give an expression of the controller \( C(s) \)
\[
C(s) = \frac{T(s)}{1 - T(s)G(s)} = \frac{1}{1 - T(s)G(s)} = \frac{1}{K \lambda s + 1 - e^{-\theta s}} \]

In the complex-frequency domain, the dead time is an irrational function. Thus \( C(s) \) can not be realised exactly. However, in the discrete domain, the dead time is approximated by the finite-dimension function. Suppose that the sampling time is \( T_s \) and \( \theta = NT_s \). By introducing a zero-order hold, we have
\[
T(z) = \frac{1}{1 - e^{-T_s/\lambda}z^{-N-1}} \]

The corresponding discrete controller can then be obtained directly from
\[
C(z) = \frac{1}{1 - e^{-T_s/\lambda}z^{-N-1}} - \frac{1}{1 - e^{-T_s/\lambda}z^{-1}} G(z) \]

We will interpret the merits of the Dahlin controller by internal model implementation. In the case of no model-plant mismatch, the classical unity feedback loop shown in Fig. 1 can be converted into the internal model control structure shown in Fig. 2 with
\[
Q(s) = \frac{G(s)}{1 + G_m(s)G(s)} \]

\[
C(s) = \frac{Q(s)}{1 - G_m(s)Q(s)} \]

where \( G_m(s) = G(\omega) \) is the model. Therefore
\[
T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)} = G(s)Q(s) \]

The structure shown in Fig. 2 is in fact an open loop.

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**Fig. 2** Structure of internal model control

The performance of the control system design is usually specified as keeping the error between the plant output \( y \) and the reference \( r \) small, or equivalently, minimising the effect of the disturbance \( d \) on the plant output \( y \). Naturally, we can take
\[
Q(s) = \frac{\tau s + 1}{K(\lambda s + 1)} \]

Then the closed-loop system transfer function is
\[
T(s) = \frac{e^{-\theta s}}{\lambda \delta + 1} \]

When \( \lambda \) tends to zero, the plant output can track the reference input exactly after the delay \( \theta \). This is in fact the result of optimal control [12].

Although the Dahlin controller is designed for a nominal plant, it is robust. This can be explained by modern robust control theory. We can describe the plant dynamic behaviour by a linear time invariant family. Assume that the plant uncertainty profile is

\[
L(\omega), \text{then the family can be represented as} \]

\[
\frac{|G(j\omega) - G_m(j\omega)|}{|G_m(j\omega)|} \leq L(j\omega) \]

From the basic result of modern robust control theory [15], it is known that the closed-loop system is stable if, and only if,

\[
||T(j\omega)G(j\omega)||_\infty < 1 \]

or equivalently

\[
||\frac{1}{\lambda j\omega + 1} e^{-\theta j\omega} L(\omega)||_\infty < 1 \quad \text{for all} \quad \omega \]

In other words, by tuning the parameter \( \lambda \), the arbitrary robust stability can be obtained for the closed-loop system.

The optimal nominal performance is usually defined as \( \min \int_0^\infty e^{\gamma t} \mathrm{d}t \) or \( \min \int W(s)S(s)\mathrm{d}s \), where \( W(s) \) is the input weighting function, and \( S(s) \) the transfer function from the input \( r \) to the error \( e \). Since the system input is a unit step, we can simply let the weight function be \( 1/s \). Observe that

\[
S(s) = \frac{1}{1 + G(s)G(s)} = 1 - T(s) \]

The robust performance can be tested by the sufficient and necessary condition as follows:

\[
\|W(s)S(s)\| + ||T(s)L(s)||_\infty = \left\| \frac{1}{j\omega}(1 - G(j\omega)Q(j\omega)) + |G(j\omega)Q(j\omega)L(\omega)| \right\|_\infty \]

\[
= \left\| \frac{1}{\lambda j\omega + 1} \right\| + \left\| \frac{1}{\lambda j\omega + 1} e^{-\theta j\omega} L(\omega) \right\|_\infty < 1 \quad \text{for all} \quad \omega \]

Hence, if the model is exact, the tuning parameter \( \lambda \) can be used for optimising the nominal performance arbitrarily. In the case of a mismatch existing between the model and the plant, performance and stability can always be evaluated by some simple conditions.

### 3 Equivalents and extensions of the Dahlin controller

In the literature one can find a very large number of digital control algorithms for single-loop systems. Each of them tries to satisfy some commonly accepted criteria. The relationship among some of them will be investigated in this paper.

#### 3.1 Dead beat control

Suppose the response of the closed-loop system to a unit step is required to equal unity at every sampling instant after the application of the unit step. If the plant is expressed as a first-order process with dead time, then the desired closed-loop model must have the pulse transfer function

\[
T(z) = z^{-N-1} \]

The controller can be obtained from the Dahlin controller with the parameter \( \lambda \rightarrow 0 \).

#### 3.2 Internal model control

The internal model control provides a framework for the design and tuning of robust controllers. It includes a two-step design procedure which was introduced by Zafiriou and Morari [13, 16]. In the first step, \( Q_m(z) \) is
designated so that no offset is produced for the given input, which means that $Q_{lm}(z)$ has to satisfy

$$Q_{lm}(z) = \frac{1}{G(z)} z^{-N-1}$$

(17)

where the subscript $^\text{\textsuperscript{--}}$ denotes the delay-free part of $G(z)$ with the zeros inside the unit circle. A mismatch between the model and the plant will generate a feedback signal which may cause performance deterioration or instability. Thus, in the second step the filter $F(z)$ is included to take care of the problem. It is of the form

$$F(z) = \frac{1}{1 - \frac{\alpha}{\alpha - 1} z^{-1}}$$

(18)

where $\alpha = e^{T_p/\lambda}, 0 \leq \alpha < 1$. For a first-order process with dead time, $Q(z)$ becomes

$$Q(z) = Q_{lm}(z)F(z)$$

$$= \frac{(1 - e^{-T_p/\lambda})(1 - e^{-T_p/\tau}z^{-1})}{K(1 - e^{-T_p/\tau})(1 - e^{-T_p/\lambda}z^{-1})}$$

and the closed-loop transfer function is given by

$$T(z) = Q_{lm}(z)F(z)G(z) = \frac{(1 - e^{-T_p/\lambda})z^{-N-1}}{1 - e^{-T_p/\lambda}z^{-1}}$$

(19)

The result is the same as that of the Dahlin controller.

3.3 One-step-ahead dynamic matrix control (DMC)

Using a step response model, Cutler and Ramaker [17] developed the algorithm. They have applied it successfully to the control of chemical processes such as in catalytic cracking units. The scheme is a typical predictive controller. It has three major features in common.

3.3.1 The finite step response model:

$$y(k) = \sum_{j=0}^{N_p-1} a_j \Delta u(k - j - 1)$$

(20)

where $\Delta$ is a difference operator and $N_p$ is the number of step response elements $a_i$ with $i \geq \theta$. The coefficients of the model can be determined by applying a unit step to the input of the process (eqn. 1).

3.3.2 The reference trajectory for the process output: Assume that $k = 1$ is the predictive horizon, $S_p$ is the set point, and $w(k)$ is the reference trajectory at time $t = k$. For many processes, a simple first-order trajectory with $i \geq \theta$ can be used

$$w(k + 1) = (1 - \alpha)S_p + \alpha w(k)$$

(21)

where $\alpha = e^{-T_p/\lambda}$. The reference trajectory can be obtained from the desired closed-loop transfer function (eqn. 2).

3.3.3 The predictive control law: The criteria function used for predictive controller design is

$$J = \left[ y(k + 1) - w(k + 1) \right]^2$$

(22)

where the symbol $^\text{\textsuperscript{*}}$ denotes estimation. Let $L$ be the control horizon and $L = P$. Then the predictive control law can be written as

$$\Delta u = A^{-1} e$$

(23)

where $A$ is the dynamic matrix and $e$ is the error between the reference trajectory and the closed-loop predictive output with zero input. It is easy to find that the result is identical with the Dahlin controller.

As we known, the classic Dahlin controller cannot be used for the control of plants with zeros outside the unit circle and unstable plants. Based on the above discussion, we will develop its extensions. Since the expressions in the discrete domain are very complex and hard to understand, only the results in the complex frequency domain are given.

3.4 Control of plants with zeros outside the unit circle

Such a plant can be expressed as

$$G(s) = K \frac{\frac{-2a_4 s + 1}{\tau s + 1}}{e^{-\theta s}} z > 0$$

(24)

The internal model control provides insights into the controller design. Take the desired closed-loop transfer function to be

$$T(s) = \frac{-2a_4 s + 1}{\tau s + 1} e^{-\theta s}$$

(25)

Then the unstable pole will be removed from the controller

$$C(s) = K \frac{\tau s + 1}{(\lambda s + 1) - (2a_4 s + 1) e^{-\theta s}}$$

(26)

3.5 Control of unstable plants

Consider the following plant

$$G(s) = \frac{1}{\tau s} e^{-\theta s}$$

(27)

which is a type II system. In order to track the step input, the closed-loop transfer function has to satisfy the following constraints

$$\lim_{s \to 0^+} [1 - T(s)] = 0$$

(28)

$$\lim_{s \to 0^+} \frac{d}{ds} [1 - T(s)] = 0$$

(29)

For stability, we can take $Q(s) = sQ_1(s)$. Then the closed-loop system is stable if, and only if, $Q_1(s)$ is stable. Let $Q_1(s)$ be of the form

$$Q_1(s) = \frac{B(s)}{(\lambda s + 1)^n}$$

where $B(s)$ is a polynomial. Combining it with eqns. 28 and 29 yields $n = 2$ and $B(s) = [2(2\lambda + \theta) s + 1]$. Then one can obtain the desired closed-loop transfer function

$$T(s) = \frac{(2\lambda + \theta) s + 1}{(\lambda s + 1)^2}$$

(30)

This results in the controller

$$C(s) = \frac{\tau s[(2\lambda + \theta) s + 1]}{(\lambda s + 1)^2 - [(2\lambda + \theta) s + 1] e^{-\theta s}}$$

(31)

4 The real cause of ringing

In discrete systems, the controller output sometimes oscillates with a period twice the sampling period. The phenomenon is known as 'ringing'. Ringing may happen in many digital controllers. The output of such a controller oscillates strongly in a manner that is often considered unsatisfactory, even although the process is being controlled as was intended. Dahlin believes that the ringing is caused by the $C(z)$'s poles near the point $z = -1$. Instead of the $C(z)$'s poles, Zhang [14] points out that the cause is $Q(z)$'s poles near the point $z = -1$. 
However, both the two formulations are ambiguous. Assume that \( \theta = NT_s + \theta' \) and \( m = 1 - \theta' / T_n \), where \( N \) is the maximum integer number of samples in the plant dead time \( \theta \). Thus \( 0 < \theta < T_s \). Utilising the modified \( z \)-transform, the discrete control can be written as

\[
G(z) = Z \left\{ \frac{1 - e^{-T_s s} K e^{-\theta t}}{s + \tau s + 1} \right\} = \frac{K}{\tau} \left[ \frac{e^{-\phi}}{s(s + \tau)} \right] - \frac{e^{-\theta s} e^{-T_s s}}{s(s + \tau)} \right] \]

\[
= \frac{K}{\tau} (1 - z^{-1}) z^{-N} \left[ \frac{e^{-\phi}}{s(s + \tau)} \right] \]

\[
= \frac{K}{\tau} (1 - z^{-1}) z^{-N} Z_m \left[ \frac{1}{s(s + \tau)} \right] \]

\[
= K (1 - z^{-1}) z^{-N-1} \frac{1}{1 - z^{-1}} \frac{1 - e^{-mT_s / \tau}}{1 - e^{-T_s / \tau} - e^{-T_s / \tau}} \]

\[
= K z^{-N-1} \frac{1}{1 - e^{-mT_s / \tau}} + z^{-1} \frac{(e^{-mT_s / \tau} - e^{-T_s / \tau})}{1 - e^{-T_s / \tau} - \tau^{-1}} \]

(32)

The desired closed-loop transfer function is as follows:

\[
T(z) = Z \left\{ \frac{1 - e^{-T_s s} e^{-\theta s}}{s + \tau s + 1} \right\} = \frac{z^{-N-1} (1 - e^{-mT_s / \lambda}) + z^{-1} (e^{-mT_s / \lambda} - e^{-T_s / \lambda})}{1 - e^{-T_s / \lambda} z^{-1}} \]

(33)

Since \( U(z) = Q(z) R(z) = T(z) R(z) G(z) \), the ringing must be caused by \( T(z) \) or \( G(z) \). It is known that the poles of \( T(z) \) are

\[
z_1 = e^{-T_s / \lambda} > 0 \quad z_2 = 0 \]

They can therefore not cause ringing. The zeros of \( G(z) \) are complex in a sense. If the dead time \( \theta \) is the integral multiple of sampling time \( T_s \), then \( G(z) \) has no zero. If the dead time \( \theta \) is not an integral multiple of \( T_s \), notice that \( 0 < e^{-T_s / \lambda} < e^{-mT_s / \tau} < 1 \), thus

\[
z_2 = \frac{e^{-T_s / \tau} - e^{-mT_s / \tau}}{1 - e^{-T_s / \tau}} < 0 \]

(34)

In this case, we have

\[
Q(z) = \frac{1 - e^{-mT_s / \lambda} + z^{-1} (e^{-mT_s / \lambda} - e^{-T_s / \lambda})}{1 - e^{-T_s / \lambda} z^{-1}} \times \frac{1}{K (1 - e^{-mT_s / \tau}) (1 - z_2 z^{-1})} \]

(35)

It is well known that the element \( (1 - z_2 z^{-1})^{-1} \), \( z_2 < 0 \), always results in ringing in discrete systems when the input is a unit step. The damping ratio of ringing is determined by the position of \( z_2 \). Detailed discussion can be found in [1].

Therefore we have the following conclusions. If the dead time \( \theta \) is an integral multiple of sampling time \( T_s \), then ringing will not occur even when there exists uncertainty in the plant model. If the dead time \( \theta \) is not an integral multiple of \( T_s \), then \( G(z) \) must have one \( z \)-plane zero lying on the negative real axis. The zero is the pole of both \( Q(z) \) and \( C(z) \). It will contribute an oscillatory mode to the controller output. Finally, the parameter \( \lambda \) is independent of ringing. When ringing exists, \( \lambda \) will affect the amplitude of ringing in a complex form, while the damping ratio is determined by the position of \( z_2 \).

5 Elimination of ringing

So far, two kinds of methods have been developed for eliminating ringing. One is proposed by Dahlin [1] and modifies the poles of the controller \( C(z) \). The other modifies the desired closed-loop response [14].

In Dahlin’s modification, to retain the useful portion of the controller output, yet eliminate ringing, the problem pole is replaced by a pole at the origin with an equivalent steady-state gain to assure transfer function settling behaviour. This can be achieved by taking \( z = 1 \) in the problem term. Although sometimes the ringing is essentially eliminated, we do not know whether any undesired effects are introduced, or not, as well as how the system performance is limited. This is why the Dahlin controller is not generally recommended in practice. The following analysis provides an interpretation for the problem.

Recall the previous discussion, \( Q(z) = C(z)(1 - G(z) C(z)) \). Thus, eliminating the problem pole of \( C(z) \) equals eliminating the problem poles of \( Q(z) \). It seems to be the case that \( \theta \) is an integral multiple of \( T_s \). However, there are two problems in Dahlin’s modification. On one hand, \( T(z) \), which is obtained when \( \theta \) is an integral multiple of \( T_s \), and \( G(z) \), which is obtained when \( \theta \) is not an integral multiple of \( T_s \), are used to derive \( C(z) \). After modifying \( C(z) \), one problem pole of \( Q(z) \) is deleted, while a new problem pole may be introduced by the term \( (1 - G(z) C(z)) \). This can be confirmed by

\[
Q(z) = \frac{1}{K} \left( \frac{1 - e^{-T_s / \lambda}}{(1 - e^{-T_s / \lambda}) (1 - e^{-T_s / \tau})} \right) \]

(36)

On the other hand, Dahlin defines \( N \) as the nearest integer to the sampling number in the dead time \( \theta \). If \( N T_s < \theta \), then the desired closed-loop response

\[
T(s) = \frac{e^{-N T_s s}}{\lambda s + 1} \]

can never be realised, also the response is not an exponential curve. If \( N T_s > \theta \), then the modified closed-loop transfer function is

\[
T(z) = \frac{1 - e^{-T_s / \lambda} (1 - e^{-mT_s / \tau}) (1 - z_2 z^{-1}) z^{-N-1}}{(1 - e^{-T_s / \lambda}) (1 - e^{-mT_s / \tau}) (1 - z_2 z^{-1}) z^{-N-1}} \]

(37)

Such a complex form makes the system analysis very difficult.

In [4] the ringing is dealt with by modifying the desired transfer function to be

\[
T(z) = \frac{1 - e^{-T_s / \lambda} z^{-N-1} (1 - z_2 z^{-1})}{1 - e^{-T_s / \lambda} z^{-N-1} z_2} \]

(38)

The problem pole of \( Q(z) \) is eliminated, as the pole of \( T(z) \) cancels the zero of \( G(z) \). Unfortunately, compared with the Dahlin controller, it is a misrepresentation and can cause the closed-loop response to be far from first order. It does not solve the same problems in Dahlin’s modification. Furthermore, the resulting controller is very complex and difficult to realise.

Based on this paper, a new procedure is developed for digital controller design, in which ringing is eliminated effectively. Define \( N \) as the maximum integer of sampling times in \( \theta \). Assume that the plant model is

\[
G(s) = \frac{K e^{-(N+1) T_s s}}{\tau s + 1} \]

(39)

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and the desired closed-loop transfer function is

$$T(s) = \frac{e^{-(N+1)T_s s}}{\lambda s + 1}$$  \hspace{1cm} (40)$$

This leads to

$$Q(z) = \frac{(1 - e^{-T_z/\lambda})(1 - e^{-T_z/\tau z^{-1}})}{K(1 - e^{-T_z/\tau})(1 - e^{-T_z/\lambda z^{-1}})}$$  \hspace{1cm} (41)$$

$$C(z) = \frac{1 - e^{-T_z/\lambda}}{1 - e^{-T_z/\lambda} z^{-1} - (1 - e^{-T_z/\lambda})z^{-N}} \times \frac{1 - e^{-T_z/\tau} z^{-1}}{K(1 - e^{-T_z/\tau})}$$  \hspace{1cm} (42)$$

Although the dead time of the plant is extended for $T_z$ in the new design procedure, no notable effects are introduced, for the dead time in the plant model is usually obtained by approximation, and $T_z$ is usually very small in practice.

**Example 1:** Suppose that the plant is described by

$$G(s) = \frac{1}{(0.5s + 1)(s + 1)^2(2s + 1)}$$

Regarding the plant model as a first-order process with dead time yields

$$G_m(s) = \frac{e^{-1.46s}}{3.34s + 1}$$  \hspace{1cm} (43)$$

Take the sampling time $T_s = 1$, then $N = 1$, $\theta = 0.46$, $m = 0.54$. The discrete model is given by an expression of the form

$$G(z) = \frac{z^{-2}(0.1439 + 0.1095z^{-1})}{1 - 0.7413z^{-1}}$$  \hspace{1cm} (44)$$

$G(z)$ has one $z$-plane zero lying on the negative real axis. There must exist ringing in the controller designed by the classical Dahlin method. The controller designed by Dahlin’s modification is

$$C(z) = \frac{1.5208(1 - 0.7413z^{-1})}{(1 - z^{-1})(1 + 0.3935z^{-1})}$$  \hspace{1cm} (45)$$

The controller proposed by [14] is

$$C(z) = \frac{1.5208(1 - 0.7413z^{-1})}{1 - 0.6065z^{-1} - 0.2271z^{-2} - 0.1667z^{-3}}$$  \hspace{1cm} (46)$$

While the new controller has the transfer function

$$C(z) = \frac{1.5208(1 - 0.7413z^{-1})}{1 - z^{-1}}$$  \hspace{1cm} (47)$$

and is the simplest. The closed-loop responses and the controller outputs are shown in Figs. 3 and 4, respectively. Both Dahlin’s modification and Zhang’s method have slow setpoint responses and large controller outputs. The superiority of the new design technique is obvious.

In particular, ringing still exists in Dahlin’s modification. Note that

$$Q(z) = \frac{1.5208(1 - 0.7413z^{-1})}{1 - 0.6005z^{-1} - 0.1724z^{-2} + 0.1667z^{-3}}$$

$$= \frac{1.5208(1 - 0.7413z^{-1})}{1 - (0.5404 + 0.2340j)z^{-1}}$$

$$\times \frac{1}{1 - (0.5404 - 0.2340j)z^{-1}(1 + 0.4804z^{-1})}$$  \hspace{1cm} (48)$$

Hence a new ringing pole is introduced. Instead of the point $z = -0.3935$, the pole is at the point $z = -0.4804$.

![Fig. 3 Responses of closed-loop systems](image)

![Fig. 4 Responses of manipulated variables](image)

### 6 Conclusions

Three main goals are achieved in this paper. The first is the interpretation of the Dahlin controller in the complex-frequency domain and the derivation of conditions that guarantee robust stability and performance for sampled-data systems. The obtained conditions can easily be checked and they provide an insight into controller tuning. The second goal is the study of the Dahlin controller and its equivalents, and their extension to the control of plants with zeros outside the unit circle or unstable plants. The desired closed-loop transfer functions and corresponding controllers are derived. The third goal is the investigation of the cause of ringing. It is found that the ringing may appear in many well known digital controllers. The essential reason is that there exist negative real zeros inside the unit circle in discrete plants. Based on the study, the limitations of conventional design techniques are analysed and a new design procedure is developed to effectively eliminate ringing.

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8 References

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